EXTENSION OF ELECTRONIC EQUIPMENT LIFE PREDICTION STUDY

FIFTH QUARTERLY PROGRESS REPORT
15 SEPTEMBER 1962 TO 15 DECEMBER 1962

Report No. 5

Contract No. DA-36-039 SC 27235

File No. 39628-PM-61-91-91 (9264)

Signal Corps Technical Requirements No. SCL-4109

Dated 19 December 1958

FOR

U.S. ARMY SIGNAL RESEARCH AND DEVELOPMENT LABORATORY, FORT MONMOUTH, NEW JERSEY

MOTOROLA INC.

MILITARY ELECTRONICS DIVISION
Western Center
8201 E. McDowell Road
Scottsdale, Arizona

FEBRUARY, 1963

WP-2643-5

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Prepared by: A. T. Kneale, Project Engineer
W. N. Simpson
M. Esher

Approved by: E. H. Lange, Section Head
Applied Research Section

Objective:

To prove the feasibility of providing field instrumentation capable of showing the rate of degradation of performance of an individual electronic equipment expressed in terms of operating time remaining until failure occurs.

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1. PURPOSE

The aim of this program is to prove the feasibility of providing field instrumentation capable of measuring the rate of degradation of performance of electronic equipment. The rate of degradation shall be measured directly on an individual electronic equipment and expressed in terms of operating time remaining until failure occurs.

The program has been divided into two phases with the phases being further reduced into specific tasks as described below:

Phase I

Task I: Design and Fabricate the Field Instrumentation

This tasks includes the selection of standard test equipment and the design and fabrication of special test equipment to be incorporated into a unified test console for measuring a selected type of field radio equipment.

Task II: Establish the Computer Program

The aim of this task is to reduce the test data into computer language and to provide the program format necessary to resolve the test data into terms of operating time remaining on individual equipment.

Task III: Obtain Test Data on a Statistically Significant Number Equipments

This task is simply to make repetitive measurements on a number of equipments of the same type, over a long period of time, to provide data for the computer.

Phase II

Task IV: Analyze Test Data to Improve Accuracy of Prediction

This task is to determine the most significant elements of test data obtained and refine the prediction process developed during Phase I.

Task V: Determine Optimum Test Data for Life Prediction on Other Types of Equipments

This task is to determine how to extend the life prediction process to other types of equipments in the field, both as to function and internal design.

Task VI: Determine Possible Methods of Testing Existing Field Equipment

This task is to evaluate physical methods of obtaining the test data required to satisfy Task V, in view of physical size and internal components used in various types of field equipments.

2. ABSTRACT

The aim of this program is to prove the feasibility of providing field instrumentation capable of measuring the rate of degradation of performance in individual electronic devices.

The major effort during the fifth quarter was as follows:

1. Develop a fresh approach to the prediction problem in order to develop a valid statistical class transition matrix $\underline{V}$.

2. Develop the programming procedure for periodically updating the embryonic transition matrix $\underline{V}$.

3. Study the possible ways in which eigenvalue solutions can be utilized in determining the entries in the unit modifier matrix $\underline{V}$.

4. Make repetitive measurements on all the AN/PRC-6 radio sets at 20-hour operating intervals, under reduced voltage stress conditions, in order to determine whether greater accuracy can be obtained from the parametric measurements, and thus a more highly developed class transition matrix $\underline{V}$ and a more accurate prediction vector $\overline{P}$.

In addition to describing the activities completed and in process, this report includes the following data:

1. The purpose of the program;

2. Breaks the program down into specific tasks;

3. The plans for the next quarter.

The program is to continue on Phases I and II until August 1963.

3. PUBLICATIONS, LECTURES, REPORTS, AND CONFERENCES

3.1 Publications

There were no publications produced during this period.

3.2 Lectures

No lectures were conducted during this quarter.

3.3 Reports

The following report concerning this project was prepared and submitted to the U.S. Army Signal Corps: EXTENSION OF ELECTRONIC EQUIPMENT LIFE PREDICTION STUDY, FOURTH QUARTERLY PROGRESS REPORT, Kneale, A.T., et al, Motorola Applied Research Section Report, WP-2643-4, 15 June 1962 to 15 September 1962.

3.4 Conferences

No conferences were held during this quarter.

4. FACTUAL DATA

The fifth Quarterly Report presents the progress of the program to date. This is presented in the following order:

1. Details on Task II: Establish the Computer Program
2. Details on Task III: Obtain Test Data on a Statistically Significant Number of Equipments.

4.1 Details on Task II; Establish the Computer Program

4.1.1 Why a Fresh Approach Was Necessary

The correlation of eigenvalues with time failed to yield any reasonable class curves as was evident and was candidly demonstrated in the fourth quarterly progress report.

In addition to the attempt at correlating the eigenvalues with time, a new method of seeking class curves was attempted which utilized the characteristic equation.

$$f(\lambda) = a_n \lambda^n + a_{n-1} \lambda^{n-1} + \ldots\ldots + a_o$$

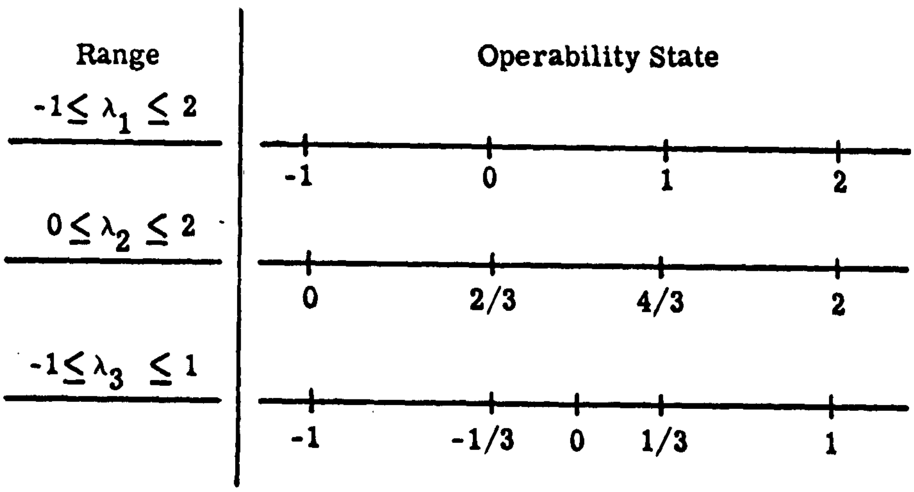
The bend points of this curve and the areas under the curve from the minimum eigenvalue to the maximum eigenvalue were plotted with time from failure and yielded no significant class curves.

Any further attempts at plotting eigenvalues with time or functions of eigenvalues with time was abandoned in favor of a completely new approach.

Before starting on any new approach to the prediction problem, it seemed reasonable to re-examine and redefine some original basic but highly important concepts.

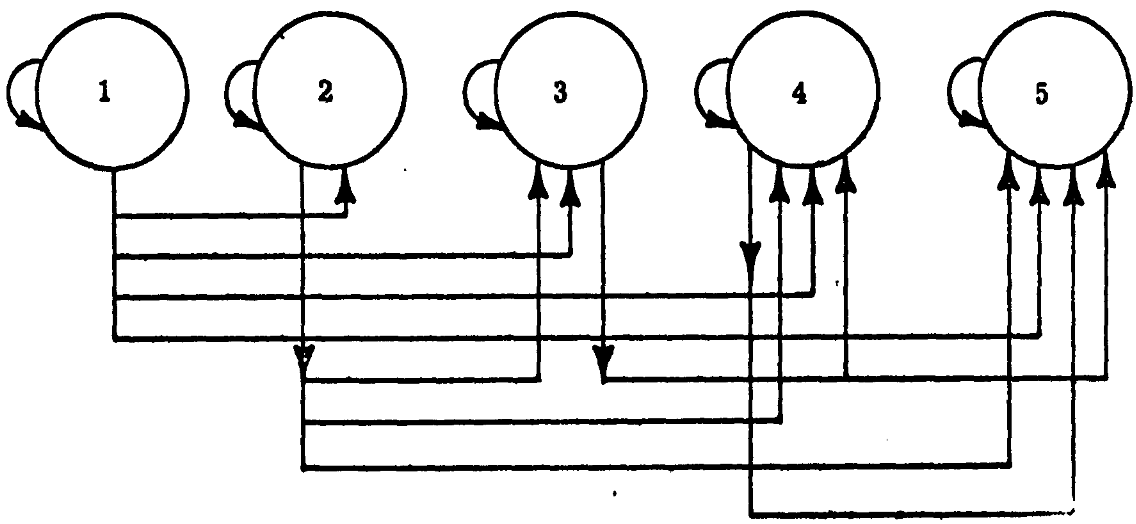
The notion of "operability state" as stated in the original theory needs a more rigorous explanation and definition, for upon the substratum of this definition must rest the implementation and outcome of the entire theory.

Upon initial application of the theory, it was thought that eigenvalues moving through their respective ranges would produce a suitable basis for the defining of operability states. The following diagram illustrates the plan for obtaining operability states from the eigenvalues for a 3 x 3 tensor matrix A.



It was then hoped that each individual of the class of electronic devices could be placed in its proper state at time, t, and a means could be obtained by which unit modifier tables could be obtained. However, such was not the outcome and a re-evaluation of the concept "operability state" was deemed necessary, in order to provide a new starting point.

According to the original theory, state reversal was not possible, since the sets were assumed to be nonself-repairable. State by-passing, however, was possible. If we assume five states, then the possible transitions are as indicated in the following diagram.



Therefore, according to the original theory, three important notions can be derived pertaining to operability states of an electronic device at time t.

1. An electronic device operating in state i at time t could stay in state i, move from state i to state j where $j = i + 1$ or move from state i to some other state k where $k > j$ in the next interval of operation.

2. It was not possible for an electronic device operating in state i to move to state p where $p < i$.

3. In addition, states were defined as follows for the operating electronic device.

   State 1 - Excellent performance

   State 2 - Good performance

   State 3 - Fair performance

   State 4 - Marginal performance

   State 5 - Failure

The three notions above were examined in view of the effort to date and in terms of what was really expected from the prediction theory.

The acceptance of (1) above could explain some of the difficulties in obtaining a class eigenvalue curve. Figure 1 illustrates a hypothetical class eigenvalue curve.

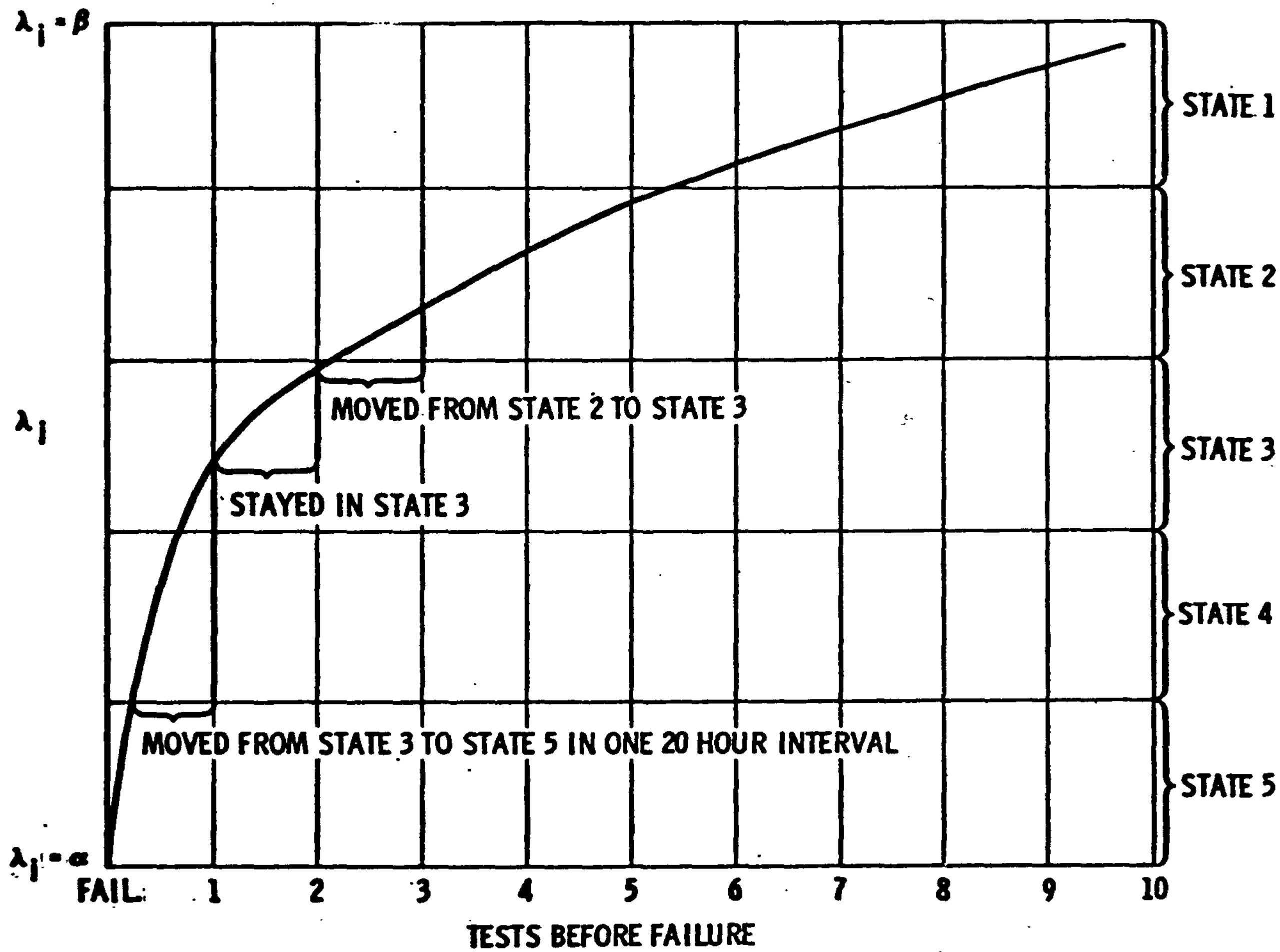


Figure 1. A Hypothetical Class Eigenvalue Curve

The $f(\lambda_i)$ would have to be a constantly increasing (or decreasing function), since state reversal was impossible.

Figure 2 represents a typical set of eigenvalue curves for various radio sets. It can be seen that the hypothetical class curve in Figure 1 is inconsistent with the actual types of eigenvalue curves obtained from the radio sets.

A class curve would assume that all sets, on the average, made the same type of state transition at $t = n$, $n = 1, 2, \ldots$ tests from failure. If Figure 1 is compared with Figure 2 at $t = 3$ and $t = 4$ tests from failure, it can readily be seen that proposition (1) above holds for Figure 2. However, Figure 1 for the class would indicate that, on the average, the entire class would tend to be in the same state at $t = 3$. Hence, it is very questionable whether it is even possible to discuss or consider a "class transition curve". However, poor quality data as was mentioned and analyzed in the fourth quarterly report would have a tremendous deterrent effect in finding a class transition curve and could invalidate the above reasoning. These are purely reflections on why class transition curves were difficult to obtain.

It is also evident from Figure 2 that state reversal occurs in many cases and, therefore, proposition (2) above does not hold.

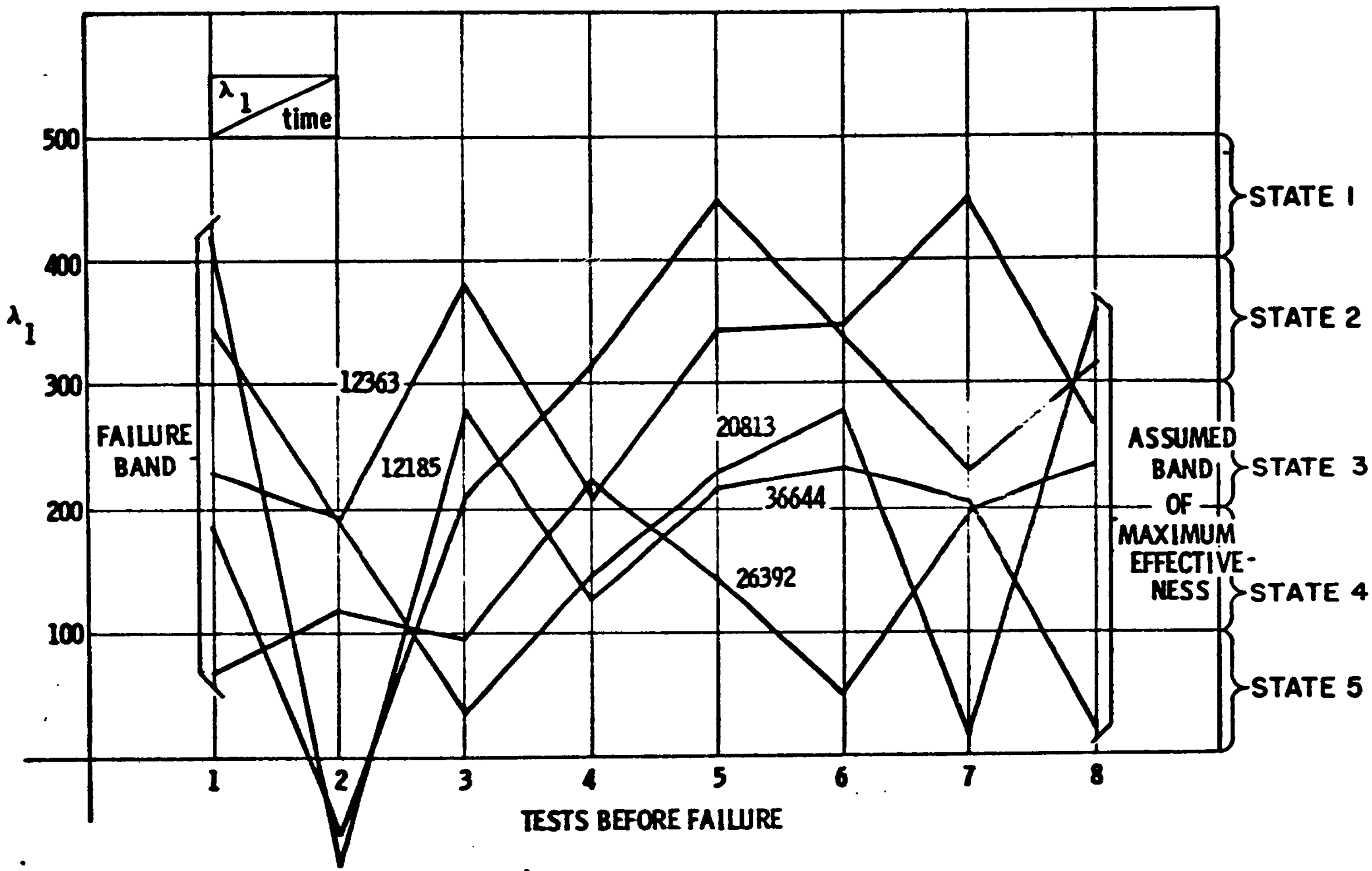


Figure 2. Eigenvalue 1 vs Time Before Failure

Defining states as in proposition (3) does not really define states at all, since the question immediately arises as to how we are to know when an entire set is in excellent operating condition, good operating condition, etc. The adjectives excellent, good, fair, make very poor indicators in a mathematical analysis.

Therefore, instead of eigenvalue ranges, a new indicator was necessary in order to define operability state. Also, mathematical meaning must be given to such relative concepts as good, excellent, fair, etc. Viewing the data with these objects in mind brings one to the conclusion that the only indicators remaining are the individual parameter values, L.O., S/N, etc. However, three important questions arise as a result of considering the parameter values as operability state indicators.

1. Is there a true maximum and minimum value for each parameter?

2. How can the range, $|\text{max.} - \text{min.}|$, be divided satisfactorily so that good, excellent, etc., be indicated?

3. How can the operating condition of the entire device be determined from the operating condition of individual parameters?

The answer to the first question is that there is a maximum and a minimum by virtue of the fact that an indicator needle on a test instrument can move through only a certain number of degrees. However, in practice,

these values are rarely achieved. So it was deemed necessary to run distributions on the parameter values at time $t = n$, $n = 1, 2, 3 \ldots, 10$ tests from failure. Observing the movement of parameter values from all radio sets in the class, optimum-maximum and minimum values were determined.

There seems to be no satisfactory agreement on how each individual range can be subdivided such that good, excellent, fair, operation can be determined. Therefore, each range was arbitrarily subdivided into ten operability states. The number of states was made deliberately large enough so that state transitions could be observed more closely. Hence, such mathematically vague words as good, excellent, fair, will be replaced by state one, state two, etc, until greater rapport exists between engineering practice and applied mathematics.

The third question is the most difficult and is not amenable to a simple solution. The question might be restated as follows: If one parameter is operating in state k, while all others are in some state below k, can we then say the entire device is in state k, since this parameter is closest to failure? If the answer is no, then we are in real difficulty indeed. If the answer is yes, then it is safe to attribute the parameter state to the state of the device.

Thus operability states are defined on the following basis:

$$\left| L.O._{max} - L.O._{min} \right| = \text{absolute range of L.O. values}$$

then

$$\frac{\left| L.O._{max} - L.O._{min} \right|}{10} = \text{increment for each operability state.}$$

Figure 3 will give the values used for operability state indicators for the receiver based on distributions run on all parameters. The two functions (receiver and transmitter) of the radio set AN/PRC-6 will be treated separately but in like manner, in order to obtain a more refined prediction.

From Figures 4 and 5 it is evident that apparent state reversal exists for individual parameters.

Figure 6 presents the transition diagram involving 10 operability states.

From the preceding discussion two new ideas have been derived.

1. A new start in defining "operability state"
2. State reversals may occur within the given data.

Figure 7 interprets Figure 6 in terms of the class transition matrix V.

Figure 7A represents the matrix obtained from the solid lines in Figure 6 and Figure 7B the matrix obtained from the dotted lines and the solid lines. The calculating of the elements in these matrices will be discussed in the next section. The matrix in Figure 7B will obviously be the one finally utilized in view of the above discussion.

State	L. O.	Fil	45	Lim	Freq	S/N	Aud.
10	2-4	40	70-77	10-13	635-638 692-695	10-12	10-17
9	4-6	41	77-84	13-16	638-641 689-692	12-14	17-24
8	6-8	42	84-91	16-19	641-644 686-689	14-16	24-31
7	8-10	43	91-98	19-22	644-647 683-686	16-18	31-38
6	10-12	44	98-105	22-25	647-650 680-683	18-20	38-45
5	12-14	45	105-112	25-28	650-653 677-680	20-22	45-52
4	14-16	46	112-119	28-31	653-656 674-677	22-24	52-59
3	16-18	47	119-126	31-34	656-659 671-674	24-26	59-66
2	18-20	48	126-133	34-37	659-662 668-671	26-28	66-73
1	20-22	49	133-140	37-40	662-668	28-30	73-80

Figure 3. Table of Operability State Values

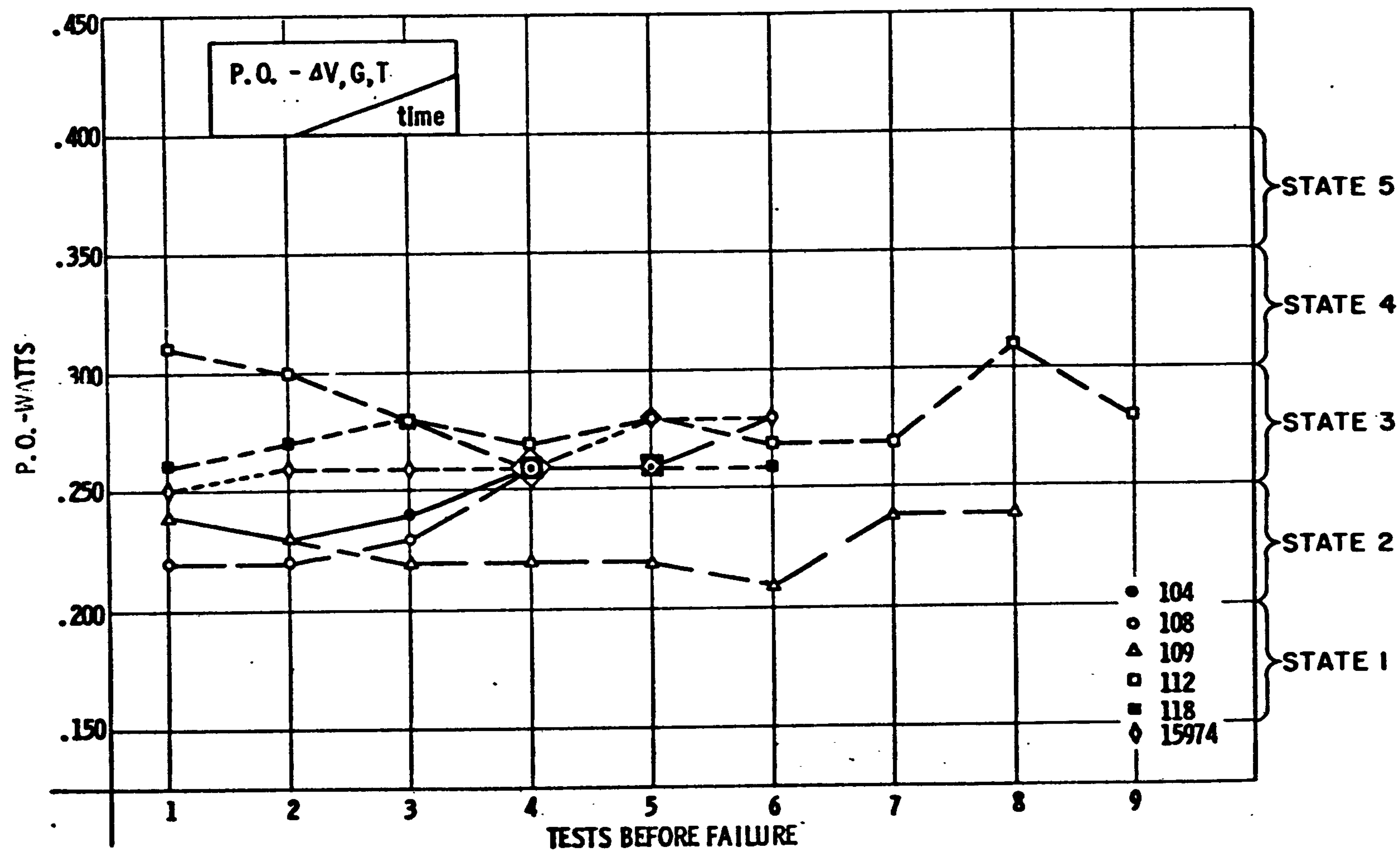


Figure 4. Power Output vs Time

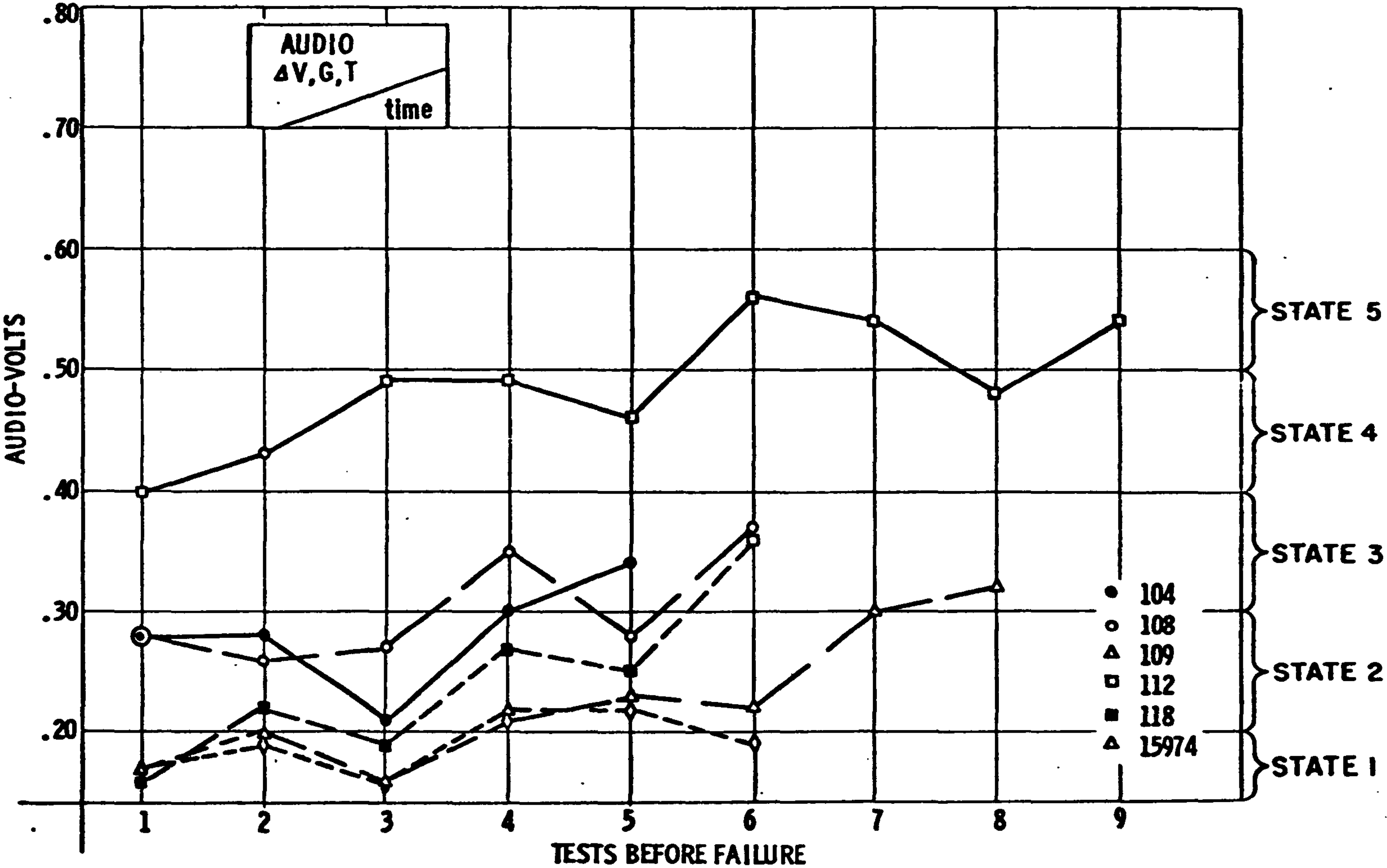


Figure 5. Audio Output vs Time

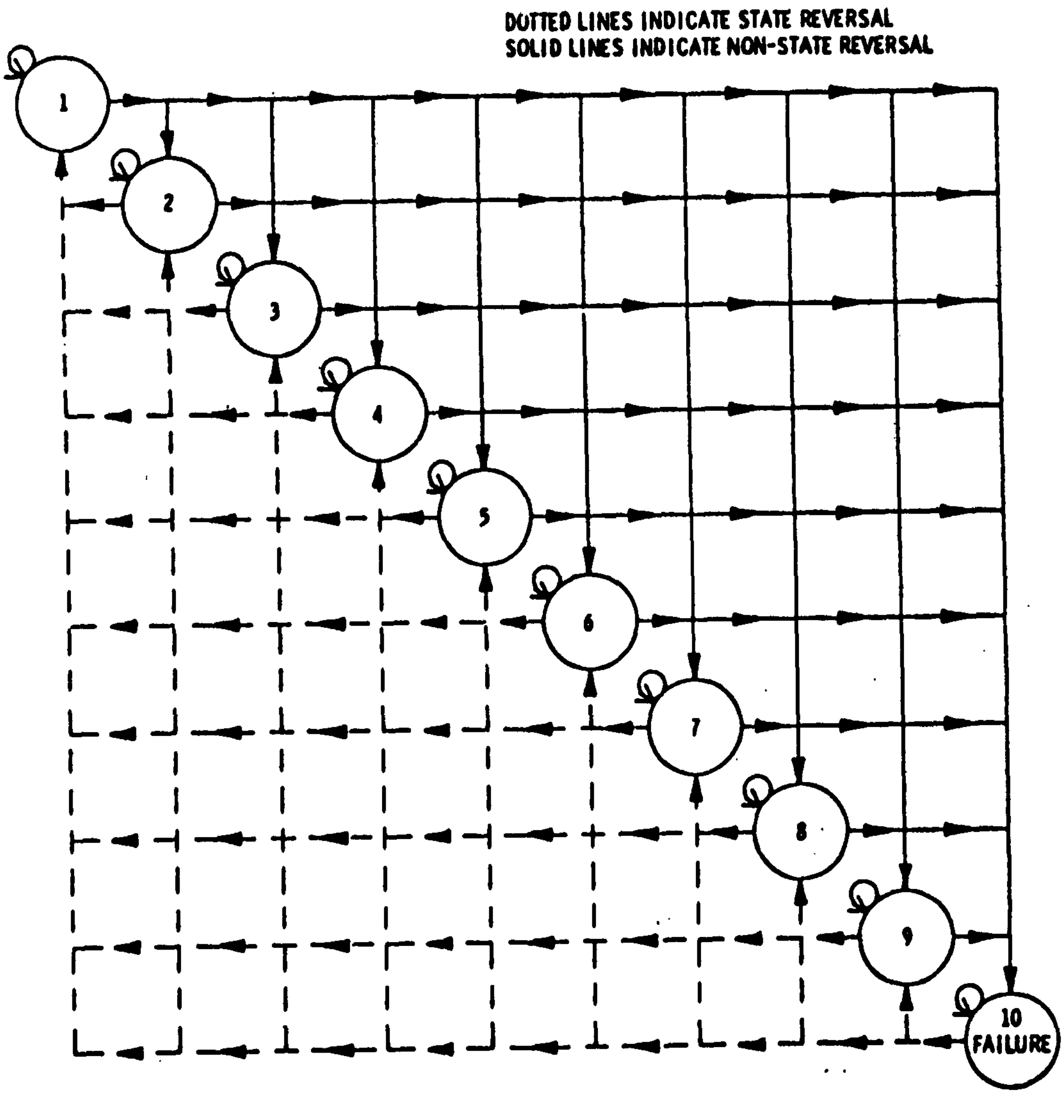


Figure 6. Operability State Transition Diagram

P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	$P_{1,10}$
0	P_{22}	P_{23}	P_{24}	P_{25}	P_{26}	P_{27}	P_{28}	P_{29}	$P_{2,10}$
0	0	P_{33}	P_{34}	P_{35}	P_{36}	P_{37}	P_{38}	P_{39}	$P_{3,10}$
0	0	0	P_{44}	P_{45}	P_{46}	P_{47}	P_{48}	P_{49}	$P_{4,10}$
0	0	0	0	P_{55}	P_{56}	P_{57}	P_{58}	P_{59}	$P_{5,10}$
0	0	0	0	0	P_{66}	P_{67}	P_{68}	P_{69}	$P_{6,10}$
0	0	0	0	0	0	P_{77}	P_{78}	P_{79}	$P_{7,10}$
0	0	0	0	0	0	0	P_{88}	P_{89}	$P_{8,10}$
0	0	0	0	0	0	0	0	P_{99}	$P_{9,10}$
0	0	0	0	0	0	0	0	0	$P_{10,10}$

A
(No State Reversibility)

P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	$P_{1,10}$
P_{21}	P_{22}	P_{23}	P_{24}	P_{25}	P_{26}	P_{27}	P_{28}	P_{29}	$P_{2,10}$
P_{31}	P_{32}	P_{33}	P_{34}	P_{35}	P_{36}	P_{37}	P_{38}	P_{39}	$P_{3,10}$
P_{41}	P_{42}	P_{43}	P_{44}	P_{45}	P_{46}	P_{47}	P_{48}	P_{49}	$P_{4,10}$
P_{51}	P_{52}	P_{53}	P_{54}	P_{55}	P_{56}	P_{57}	P_{58}	P_{59}	$P_{5,10}$
P_{61}	P_{62}	P_{63}	P_{64}	P_{65}	P_{66}	P_{67}	P_{68}	P_{69}	$P_{6,10}$
P_{71}	P_{72}	P_{73}	P_{74}	P_{75}	P_{76}	P_{77}	P_{78}	P_{79}	$P_{7,10}$
P_{81}	P_{82}	P_{83}	P_{84}	P_{85}	P_{86}	P_{87}	P_{88}	P_{89}	$P_{8,10}$
P_{91}	P_{92}	P_{93}	P_{94}	P_{95}	P_{96}	P_{97}	P_{98}	P_{99}	$P_{9,10}$
$P_{10,1}$	$P_{10,2}$	$P_{10,3}$	$P_{10,4}$	$P_{10,5}$	$P_{10,6}$	$P_{10,7}$	$P_{10,8}$	$P_{10,9}$	$P_{10,10}$

B
(State Reversibility)

Figure 7. Class Transition Matrices

4.1.2 Development of the Class Transition Matrix V from the Raw Data.

Figure 8 denotes the completion of the data processing to date. Note the barrier which exists and still exists between blocks (4) and (5). Since the barrier to date has been insurmountable, it seemed logical to apply the information obtained from the foregoing analysis in 4.1.1 toward efforts to complete another block of the prediction process. Block (7), the Class Transition Probability Matrix V was chosen, since it seemed to flow most easily from the new basis for operability states.

4.1.2.1 Probability Theory Utilized in Forming the Embryonic Class Transition Matrix V.

A section exists in the Second Quarterly Progress Report dated 15 October 1961 to 15 January 1962 titled "4.2.5 Class Transition Probability Matrix 'Statistical' " which deals with the method of formation of the class transition matrix V. This approach will be superceded by an approach contiguous to the basic theory as presented in the Final Progress Report, dated 1 July 1959 to 31 December 1960.

There were many objections to the former approach, three of which follow:

(1) The matrix was not based on the original data but upon a class probability curve.

(2) The curve was improperly interpreted.

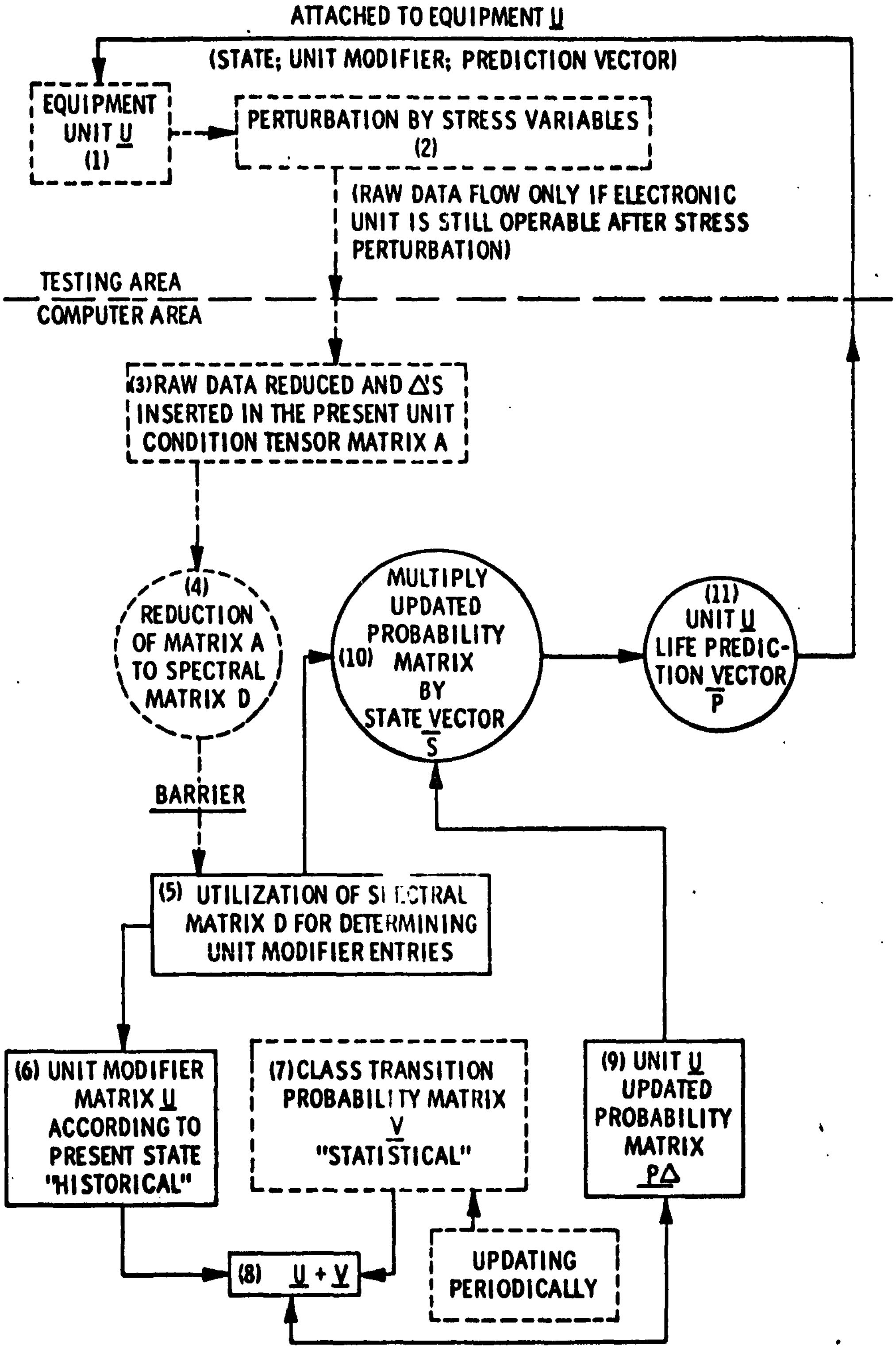


Figure 8. Block Diagram, Completion of Data Processing to Date

(3) Once the matrix was obtained, it was not amenable to updating. These three difficulties will be completely overcome in the following approach.

The rule for calculating calculable probabilities is very nicely stated by William Burnside (1).

"Rule. The results of a trial or choice, or the trial itself, or both the trial and the results, are subject to such conditions that, wherever, whenever, and by whomever the trial is made, there are just n possible results, of which one must occur and only one can occur. If in n_A of these results the condition A is satisfied, while in the remaining $n - n_A$ it is not satisfied, the probability that the condition A is satisfied, when a trial is made, is $\frac{n_A}{n}$; provided that each two of the n results are assumed to be equally likely."

The underlined statement can also read: provided that, for each condition A, the n_A results which satisfy condition A are assumed to be equally likely.

The word "trial" in the rule above is equivalent to test and the word "condition" is equivalent to operability state for calculating the transition probabilities in the class transition matrix $\underline{V}$.

It is also stated, "If a trial is repeated, and it is proposed to consider probabilitiies connected with the repeated trial, it is necessary to make an assumption of equal likelihood. Suppose there are N possible results

each two of which are equally likely for the repeated trial, and that in Nij of them, the ith result occurs at the first trial and the jth at the second. For the repeated trial, subject to the condition that the ith result occurs in the first, there are just

$$Ni_1 + Ni_2 + \dots + Ni_n$$

results; and each two of them are equally likely. The result of the first trial is not relevant to the second, so that the probability that the jth result occurs at the second trial is

$$Pij = \frac{Nij}{\sum_{j=1}^{n} Nij}, \quad i = 1, 2, 3, \dots, n$$

From Feller (2) the following definition is given with modifications to elucidate its application to the life prediction problem.

"<u>Definition.</u> A sequence of tests at 20-hour intervals with possible outcomes state K, state L, ... will be called a Markov chain, if the probabilities of sample test sequences are defined as:

$$P\left\{\text{state K, state L, state R, } \dots \text{, state n}\right\} =$$

$$P_{KK}\,P_{KL}\,P_{LR} \cdots P_{mn}$$

in terms of an initial probability distribution P_{KK} for the state K at time $n = 1$ and fixed conditional probabilities P_{LR} of state R, given that state L has occurred at the preceding trial.

Instead of saying the nth test results in state k, we shall say at time n the system is in state k. The conditional probability will be called the probability of the transition $j \rightarrow k$ (from state j to state k).

The transition probabilities will be arranged in a matrix of transition probabilities

$$\underline{V} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & \cdots \\ P_{21} & P_{22} & P_{23} & \cdots \\ P_{31} & P_{32} & P_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

where the first subscript stands for row, the second for column. Clearly, $\underline{V}$ is a square matrix with non-negative elements and unit row sums. Such a matrix is called a stochastic matrix. Any stochastic matrix can serve as a matrix of transition probabilities; together with out initial distribution

$\left\{ P_{KK} \right\}$ it completely defines a Markov chain with states K, L,

. . . .

4.1.2.2 A Method for Obtaining the Elements Pij in a 10 x 10 Class Transition Matrix V (Embryonic).

Two matrices were developed according to the following criteria:

(1). Time was considered and thus the number of tests from failure was kept track of in the development of the matrix.

(2) The matrix was developed by considering all trials in terms of consecutive pairs $t = i$ to $t = i + 1$. This approach frees us of time considerations which were seen to be a barrier in the preceding analysis.

Although the matrix in (2) above will be the one finally relied upon, the matrix in (1) gives an indication of class behavior and also gives another indication of the reliability of the data. A further subdivision exists if we develop matrices on the assumption of non state-reversal and also on the proven fact that they do reverse in actuality (according to our definition of operability state and keeping in mind the quality of the data). All four matrices were developed as follows:

(1) A Development of the matrix from time considerations and from state reversibility utilizing single parameter values.

Figure 3 contains the operability state boundary values. The number of parameter values N_1 in state 1 at 10 tests from failure and their actual positions in the raw data matrix are recorded. These positions are then examined in the raw data card for nine tests until failure. The number N_{11} that stayed in state 1, the number N_{12} that moved from 1 to 2, etc is then recorded. Then, the number of parameter values N_2 in state 2 at nine tests from failure is recorded and their actual positions in the raw data matrix. These positions are then examined in the raw data card for eight tests from failure. The number N_{21} that moved from state 2 to state 1, the number N_{22} that stayed in state 2, etc is then recorded. The above procedure is continued until the number and positions of values at 1 test from failure are tracked to the actual failure data card. Figures 9 and 10 should clarify the above procedure. Only data from sets which have failed can be utilized in this procedure. This restriction, however, will be later overcome.

The transition matrix elements are then calculated as follows from Figure 10.

t = 10 TESTS FROM FAILURE NO. 1-2

TEST CARD AN/PRC-6

DATE 4/11/62 SERIAL NO. 114 TEST POSITION A OR B (CIRCLE ONE)

	RECEIVE							TRANSMIT						
	1 L.O.	2 FIL	3 45	11 LIM	12 FREQ	13 S/N	14 AUDIO	4 PAG	5 PO	6 FIL	7 45	8 90	9 FREQ	10 Δ F
V, G, T	20	44	129	30	684	[illegible]	40							
ΔV, G, T	17	42	100	24	682	16	26							
ΔV, ΔG, T														
V, ΔG, T	20	44	129	29	678	18	42							
V, G, ΔT	20	44	129	29	672	19	41							
ΔV, G, ΔT														
ΔV, ΔG, ΔT														
V, ΔG, ΔT														

→ VALUES IN STATE ONE*

t = 9 TESTS FROM FAILURE NO. 1-3

TEST CARD AN/PRC-6

DATE 4/19/62 SERIAL NO. 114 TEST POSITION A OR B (CIRCLE ONE)

	RECEIVE							TRANSMIT						
	1 L.O.	2 FIL	3 45	11 LIM	12 FREQ	13 S/N	14 AUDIO	4 PAG	5 PO	6 FIL	7 45	8 90	9 FREQ	10 Δ F
V, G, T	22	45	129	27	678	14	35							
ΔV, G, T	17	42	104	20	678	13	22							
ΔV, ΔG, T														
V, ΔG, T	22	45	129	27	678	13	35							
V, G, ΔT	22	45	129	27	677	12	35							
ΔV, G, ΔT														
ΔV, ΔG, ΔT														
V, ΔG, ΔT														

→ VALUES ALL REMAINED IN STATE ONE*

*RINGED VALUES COULD BE UNDER ANY PARAMETER

Figure 9. Sample Test Sequence Noting State Transitions

N_i, NUMBER OF ELEMENTS IN STATE i AT TIME t	NUMBER OF ELEMENTS MAKING TRANSITIONS FROM STATE i TO STATE j AT TIME t + 1 STATE j									
	1	2	3	4	5	6	7	8	9	10
N_i @ 10 TESTS BEFORE FAILURE = 9	6	3	0	0	0	0	0	0	0	0
N_2 @ 9 = 16	5	6	3	0	0	2	0	0	0	0
N_3 @ 8 = 52	4	11	27	3	7	0	0	0	0	0
N_4 @ 7 = 81	0	1	12	38	30	0	0	0	0	0
N_5 @ 6 = 158	0	0	3	23	80	45	5	2	0	0
N_6 @ 5 = 188	10	1	4	14	44	96	12	5	1	1
N_7 @ 4 = 80	0	0	0	0	4	8	39	26	1	2
N_8 @ 3 = 116	0	0	0	0	1	9	33	54	18	1
N_9 @ 2 = 68	0	0	0	0	1	1	8	16	33	9
N_{10} @ 1 = 35	0	0	0	0	0	0	1	2	6	26

Figure 10. Distribution of Elements Making State Transitions

$$P_{ij} = \frac{N_{ij}}{\sum_{K=1}^{10} N_{ik}} = \frac{N_{ij}}{N_i}$$

Example:

$$P_{11} = \frac{N_{11}}{N_1} = \frac{6}{9} = 0.667$$

$$P_{12} = \frac{N_{12}}{N_1} = \frac{3}{9} = 0.333$$

The completed embryonic class transition matrix $\underline{V}$ is presented in Figure 11. From observation of the values in the matrix, the analysis in 4.1.1 appears to be correct. However, one must be careful in the interpretation of the matrix in Figure 11. The following interpretation should make this clear. If a parameter value resides in state 1 at 10 tests before failure then the probability that this element will stay in state 1, move to state 2, etc at nine tests from failure is P_{11} =0.667, P_{12} =0.333, etc.

This matrix reveals where key parameter values lie at one test from failure and also indicates strong state reversibility. It also indicates a strong probability that an element in state i at time t will stay in state i at t + 1. Remember, however, this is an embryonic matrix based on an insufficient quantity of data. By updating periodically this should become a more accurate statistical matrix.

@ 10 Before Failure	.667	.333	0	0	0	0	0	0	0	0
@ 9	.313	.375	.188	0	0	.125	0	0	0	0
@ 8	.077	.212	.519	.058	.135	0	0	0	0	0
@ 7	0	.012	.148	.469	.370	0	0	0	0	0
@ 6	0	0	.019	.146	.506	.285	.032	.006	0	0
@ 5	.053	.005	.021	.074	.234	.511	.064	.027	.005	.005
@ 4	0	0	0	0	.050	.100	.488	.325	.013	.025
@ 3	0	0	0	0	.009	.078	.284	.466	.155	.009
@ 2	0	0	0	0	.015	.015	.118	.235	.485	.132
@ 1	0	0	0	0	0	0	.029	.057	.171	.743

Figure 11. Class Transition Matrix V; State Reversal Allowed Developed from Single Parameter Values

(1) B <u>Development of the matrix from time considerations, assuming nonreversibility of states and utilizing single parameter values.</u>

The elements in this matrix are determined as follows from Figure 10.

$$N_i - \sum_{\substack{K = 1 \\ i > K}}^{K = i - 1} N_{ik} = N'_i$$

$$P'_{ij} = \frac{N_{ij}}{N_i - \sum\limits_{\substack{k = 1 \\ i > K}}^{k = i - 1} N_{ik}} = \frac{N_{ij}}{N'_i}$$

<u>Example:</u>

$$P'_{33} = \frac{N_{33}}{N_3 - \sum\limits_{j = 1}^{2} N_{3j}} = \frac{27}{52 - (4 + 11)} = \frac{}{52 - (15)} = \frac{27}{37} = 0.730$$

$$P'_{34} = \frac{N_{34}}{N_3 - \sum\limits_{j = 1}^{2} N_{3j}} = \frac{3}{37} = 0.081$$

Figure 12 presents this matrix.

Although the matrix in Figure 11, as a stochastic transition matrix, gives insights into class parameter behavior, and gives apparent verification of the preceding analysis, it is still deficient in certain respects as far as

@ 10 Before Failure	.667	.333	0	0	0	0	0	0	0	0
@ 9	0	.545	.273	0	0	.182	0	0	0	0
@ 8	0	0	.730	.081	.189	0	0	0	0	0
@ 7	0	0	0	.559	.441	0	0	0	0	0
@ 6	0	0	0	0	.611	.344	.038	.008	0	0
@ 5	0	0	0	0	0	.835	.104	.043	.009	.009
@ 4	0	0	0	0	0	0	.574	.382	.015	.029
@ 3	0	0	0	0	0	0	0	.740	.247	.014
@ 2	0	0	0	0	0	0	0	0	.786	.214
@ 1	0	0	0	0	0	0	0	0	0	1

Figure 12. Class Transition Matrix V; No State Reversal Developed from Single Parameter Values

the prediction process is concerned. The following deficiencies still exist in this matrix;

1. It involves the element of time;
2. Many devices operate longer than ten tests;
3. It gives probability information for parameters only and doesn't indicate transition probabilities for the entire device.
4. <u>All</u> parameter elements in <u>every</u> test are not considered.

These four deficiencies will be overcome in the following approach.

(2) A <u>Development of the matrix free of time considerations and allowing state reversibility</u>.

The parameters filament and frequency were removed from the receiver test card, since their performance was highly erratic.

If only two consecutive tests are considered, time considerations can be completely eliminated and <u>any</u> two consecutive tests can be used to update the basic transition matrix $\underline{V}$. All elements (except filament and frequency) on every test card will be utilized in forming the matrix elements and, thus, probability information can be obtained for the entire device and not solely for parameters.

A new operability state range table was developed in order to detect state transitions more readily. Figure 13 contains this table. These values are always amenable to change without affecting the basic procedure.

OPERABILITY STATE	L.O.	45	LIM	S/N	AUD.
10	2-5	85-90	10-13	10	10-16
9	5	90-95	13-16	11	16-20
8	6	95-100	16-18	12	20-24
7	7	100-103	18-20	13	24-26
6	8	103-106	20	14	26-30
5	9	106-108	21	15	30-34
4	10	108-110	22	16	34-36
3	11	110-112	23	17	36-38
2	12	112-114	24	18	38-40
1	≥ 13	≥ 114	≥ 25	≥ 19	≥ 40

Figure 13. Table of Revised Operability State Values

All pertinent values in every data card had to be interrelated, in order to determine the operability state of the device and not merely a parameter. All four values under every parameter were averaged and, of these five averages, the one being in a state closest to failure determined the operability state of the device at the time the data was recorded. Of course, the assumption enters here that the state of the device depends on the state of a single parameter. Figure 14 illustrates the method for determining the operability state of a single device at time $t = 1$.

The operability state of a device at time $t = n$ is recorded and the transition at time $t = n + 1$ for the same device is also recorded. Continuation in this fashion produces the required values for development of the true class transition matrix $\underline{V}$. These values are defined as follows:

T_i = Total number of sets in state i at $t = n$, $i = 1, 2, \ldots, 10$.

T_{ij} = Total number of sets in state i at $t = n$ and making transitions to state j at $t = n + 1$, i and $j = 1, 2, \ldots, 10$.

Figure 15 presents one of ten tables necessary in forming the transition probabilities. The conditional probabilities are then defined

$$P_{ij} = \frac{T_{ij}}{T_i}; \quad \sum_{j=1}^{10} P_{ij} = 1, \; i = 1, 2, \ldots, 10$$

6 TESTS FROM FAILURE

TEST CARD AN/PRC-6

NO. 1-1

DATE 4/10/62 SERIAL NO. 108 TEST POSITION A OR B (CIRCLE ONE)

	RECEIVE							TRANSMIT						
	1 L.O.	2 FIL	3 45	11 LIM	12 FREQ	13 S/N	14 AUDIO	4 PAG	5 PO	6 FIL	7 45	8 90	9 FREQ	10 F
V, G, T	7		111	30		16	31							
ΔV, G, T	5		87	15		11	9							
ΔV, ΔG, T														
V, ΔG, T	7		111	30		16	31							
V, G, ΔT	6		112	31		16	31							
ΔV, G, ΔT														
ΔV, ΔG, ΔT														
V, ΔG, ΔT														

AVERAGE L.O. $\frac{7+5+7+6}{4}$ = 6 LIES IN STATE 8 (FROM FIG. 13)

AVERAGE 45 $\frac{111+87+111+112}{4}$ = 105 LIES IN STATE 6

AVERAGE LIM $\frac{30+15+30+31}{4}$ = 27 LIES IN STATE 1

AVERAGE S/N $\frac{16+11+16+16}{4}$ = 15 LIES IN STATE 5

AVERAGE AUDIO $\frac{31+9+31+31}{4}$ = 26 LIES IN STATE 6

THE RADIO SET IS THEN SAID TO BE IN STATE 8

Figure 14. Determination of Operability State

Time n	Number of Sets in State 5	State Transitions of Sets at Time t = n+1										Time n + 1
		1	2	3	4	5	6	7	8	9	10	
17 before failure	0											16
16	0											15
15	1		1									14
14	0											13
13	0											12
12	3		1	1		1						11
11	3					2	1					10
10	3					2		1				9
9	3					2		1				8
8	5			2		2		1				7
7	4			2	1		1					6
6	2					1				1		5
5	9	1			2	3	1	2				4
4	4					1	1		1	1		3
3	6		2		1	2			1			2
2	11				1	1	2	5	1	1		1
1	8						1	1			6	Fail.
Totals	$T_5 = 62$	$T_{51} = 1$	$T_{52} = 4$	$T_{53} = 5$	$T_{54} = 5$	$T_{55} = 17$	$T_{56} = 7$	$T_{57} = 11$	$T_{58} = 3$	$T_{59} = 3$	$T_{5,10} = 6$	

Figure 15. Sample Distribution of Radio Sets Making State Transitions

Example:

$$P_{51} = \frac{T_{51}}{T_5} = 1/62 = .016$$

Figure 16 contains the final class transition matrix $\underline{V}$, allowing for state reversibility. It can readily be seen that it removes the discrepancies found in the matrix developed in 1A above (Figure 11).

(2) B Development of the matrix free of time considerations and assuming non-state reversibility.

The entries in this matrix are obtained from the following relation.

$$P'_{ij} = \frac{T_{ij}}{T_i - \sum_{K=1}^{K=j-1} T_{iK}}, \quad i \text{ and } j = 1, 2, \ldots, 10$$

Example:

$$P'_{56} = \frac{T_{56}}{T_5 - \sum_{K=1}^{K=4} T_{5K}} = \frac{7}{62 - (15)} = \frac{7}{47} = .149$$

Figure 17 presents this matrix. The matrix in Figure 16 will be the one actually used in the prediction process. It is stored in memory and updated periodically. It is felt that this matrix is the one most accurately representing set transitions. The computer flow diagram for updating this matrix is presented in Figure 18.

.375	.250	.125	.156	.031	0	.031	0	0	.031
.233	.200	.200	.100	.133	0	.067	0	0	.067
.111	.056	.167	.333	.056	.167	.056	0	0	.056
.054	.071	.161	.304	.214	.125	.018	.036	0	.018
.016	.065	.081	.081	.274	.113	.177	.048	.048	.097
0	.017	0	.100	.150	.433	.133	.067	0	.100
0	0	0	.019	.074	.093	.259	.204	.111	.241
0	0	0	.054	.027	.027	.216	.459	.162	.054
0	0	0	0	.043	.043	.130	.087	.348	.348
0	0	0	0	.167	0	0	0	0	.833

Figure 16. Class Transition Matrix V; Time Independent, State Reversal

.375	.250	.125	.156	.031	0	.031	0	0	.031
0	.261	.261	.130	.174	0	.087	0	0	.087
0	0	.200	.400	67	.200	.067	0	0	.067
0	0	0	.425	.300	.175	.025	.050	0	.025
0	0	0	0	.362	.149	.234	.064	.064	.128
0	0	0	0	0	.591	.182	.091	0	.136
0	0	0	0	0	0	.318	.250	.136	.295
0	0	0	0	0	0	0	.680	.240	.080
0	0	0	0	0	0	0	0	.500	.500
0	0	0	0	0	0	0	0	0	1

Figure 17. Class Transition Matrix V; Time Independent; No State Reversibility

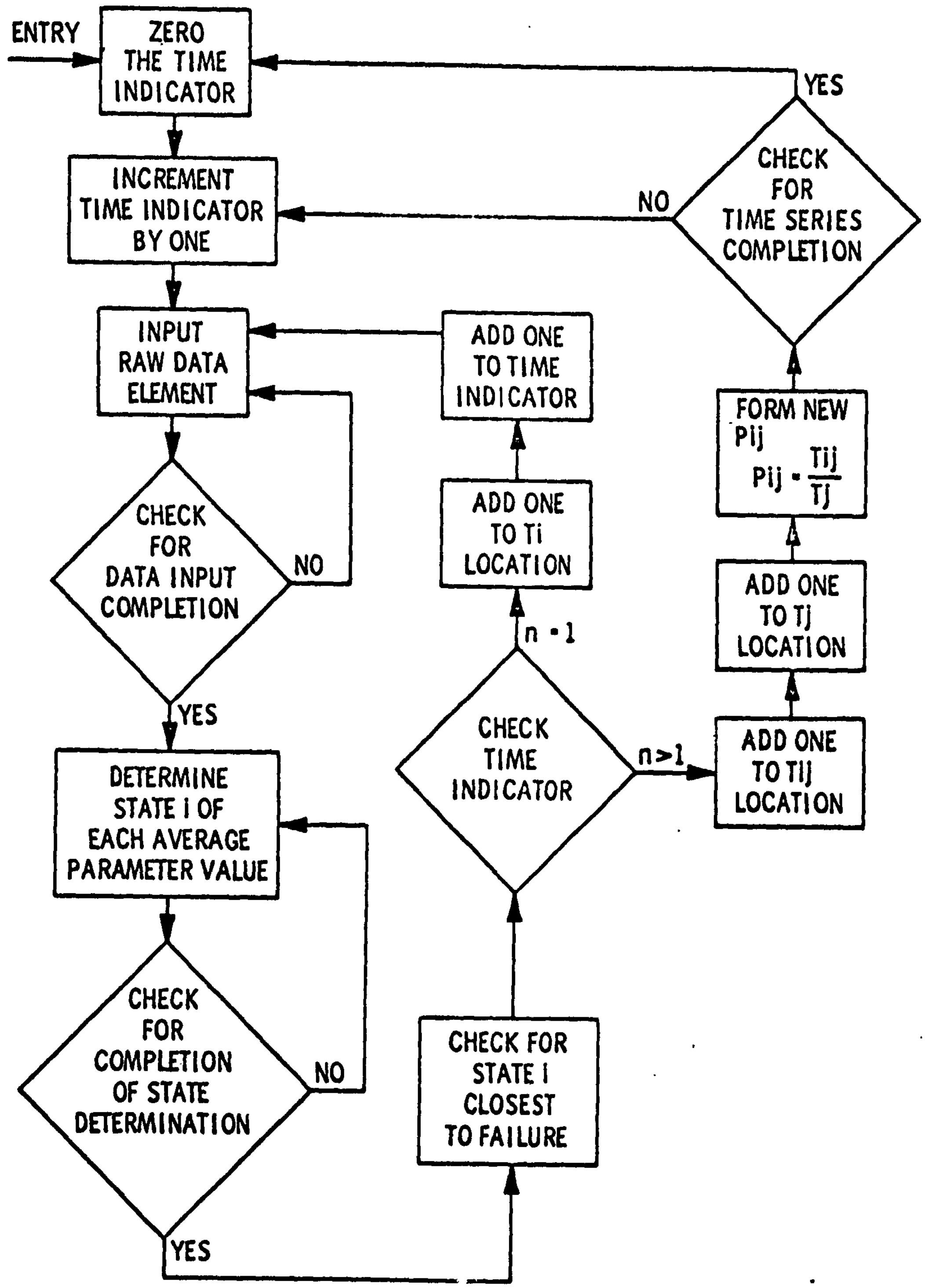


Figure 18. Computer Flow Diagram For the Updating of V

A study is now under way to determine the feasibility of utilizing eigenvalue solutions in the development of unit modifier table entries. If these attempts prove fruitful, the barrier indicated in Figure 8 may be surmounted.

4.2 Details on Task III: Obtain Test Data on a Statistically Significant Number of Equipments.

Testing of the AN/PRC-6 radio sets is continuing at the same rate as before. The reference stress level of the voltage has been shifted down to the lower limit of the radio set design limit from the center of the design criteria. This has been done to determine whether the strain tensor might be more sensitive at this point than in the center. The present analysis procedure will be applied to this data as soon as practical. Of course a new embryonic class transition matrix $\underline{V}$ must be developed in order to indicate class behavior under the lower stress conditions.

5. CONCLUSIONS

From an analysis of what has been accomplished prior to the Fourth Quarterly Progress Report, it was deemed necessary to attack the prediction problem from a different standpoint.

A re-evaluation of the meaning of operability state was made and operability states were redefined in a manner more amenable to a mathematical analysis. This time, the notion of operability state was based upon parameter ranges and not upon eigenvalue ranges.

A successful class transition matrix was obtained which is free of time considerations and indicates set transitions. A means for updating this matrix can be readily achieved.

Testing of the AN/PRC-6 radio sets is continuing at the same rate as before. The reference stress level of the voltage has been shifted down to the lower limit of the radio set design limit from the center of the design criteria. This has been done to determine whether the strain tensor might be more sensitive at this point than in the center.

6. PROGRAM FOR NEXT QUARTER

A careful analysis of the tensor matrix A and its resultant spectral matrix D of eigenvalues must be made in order to determine whether it is possible to utilize the matrix D in the determination of unit modifier table entries. This is the final barrier which must be surmounted in order to achieve a successful implementation of the mathematical model for failure prediction.

Of course, one cannot assume that a successful implementation will bring successful results. At best, one can only hope for a high percentage of successful predictions, since the final output vector will be in terms of probability, and thus an element of uncertainty always exists.

Thus, the following two objectives will be considered in project efforts during the next quarter in the necessary order in which they must occur.

1. Successfully complete implementation of the mathematical model.

2. Actually make predictions in order to determine percentage accuracy.

7. IDENTIFICATION OF PERSONNEL

The efforts of the following technical personnel were expended on this project during fourth period. No new technical personnel were assigned to this project during the period.

Name	Title	Hours
A. T. Kneale	Project Leader	178
M. Esher	Electrical Engineer	624
W. N. Simpson	Mathematician	944
E. Lange	Applied Research Section Head	Consultant
K. W. Porter	Chief, Engineer, Telecommunications Laboratory	Consultant

APPENDIX

BIBLIOGRAPHY

(1) Burnside, William, Theory of Probability, Dover Publications, New York, 1959.

(2) Feller, W. A., An Introduction To Probability Theory and Its Applications, New York, 1950.